Characterizing Non-Classicality and Entanglement in Generalized Coherent States:

Bridging Theoretical Foundations and Experimental Criteria

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Summer Internship of Physics 2026

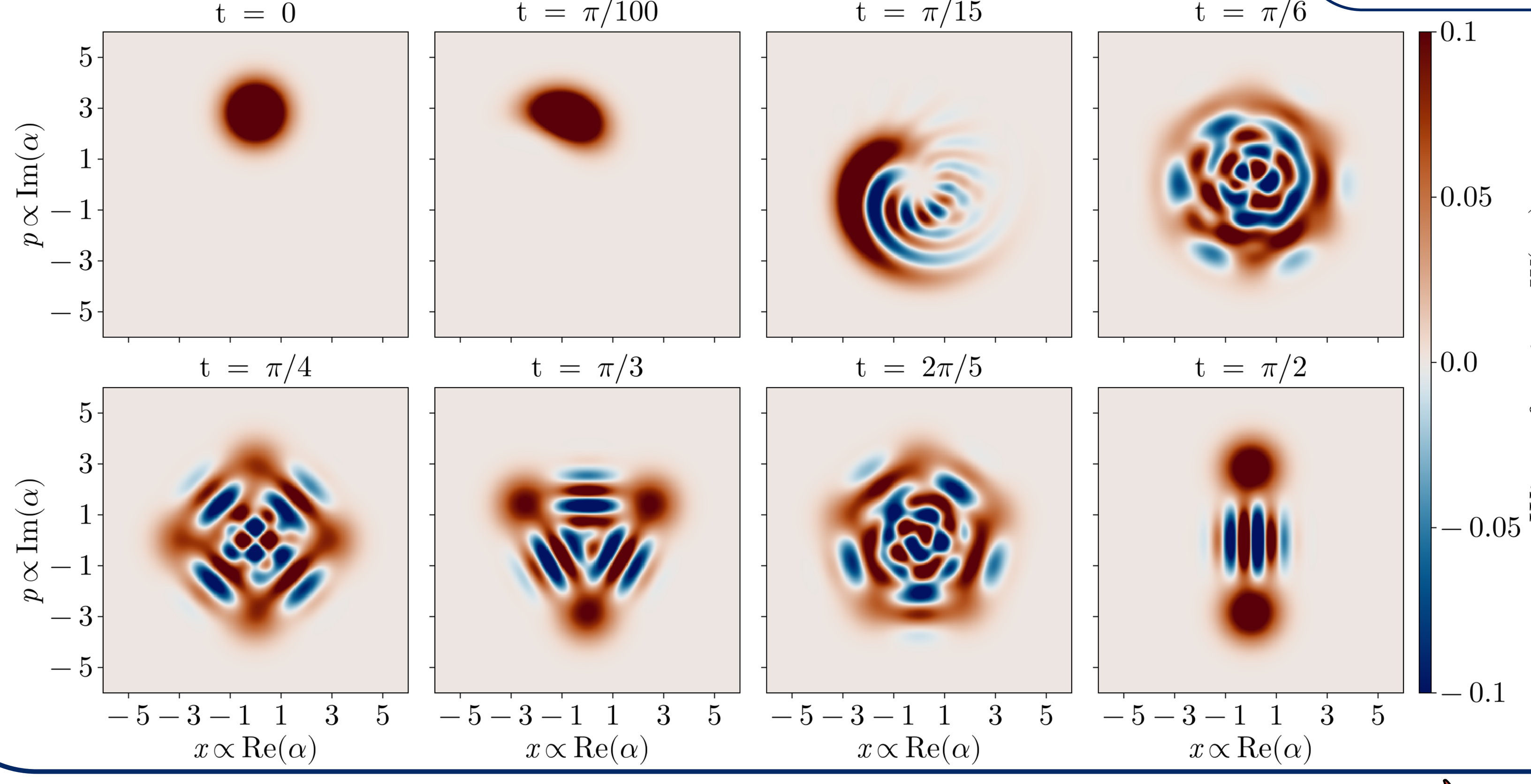
Quasiprobability distributions

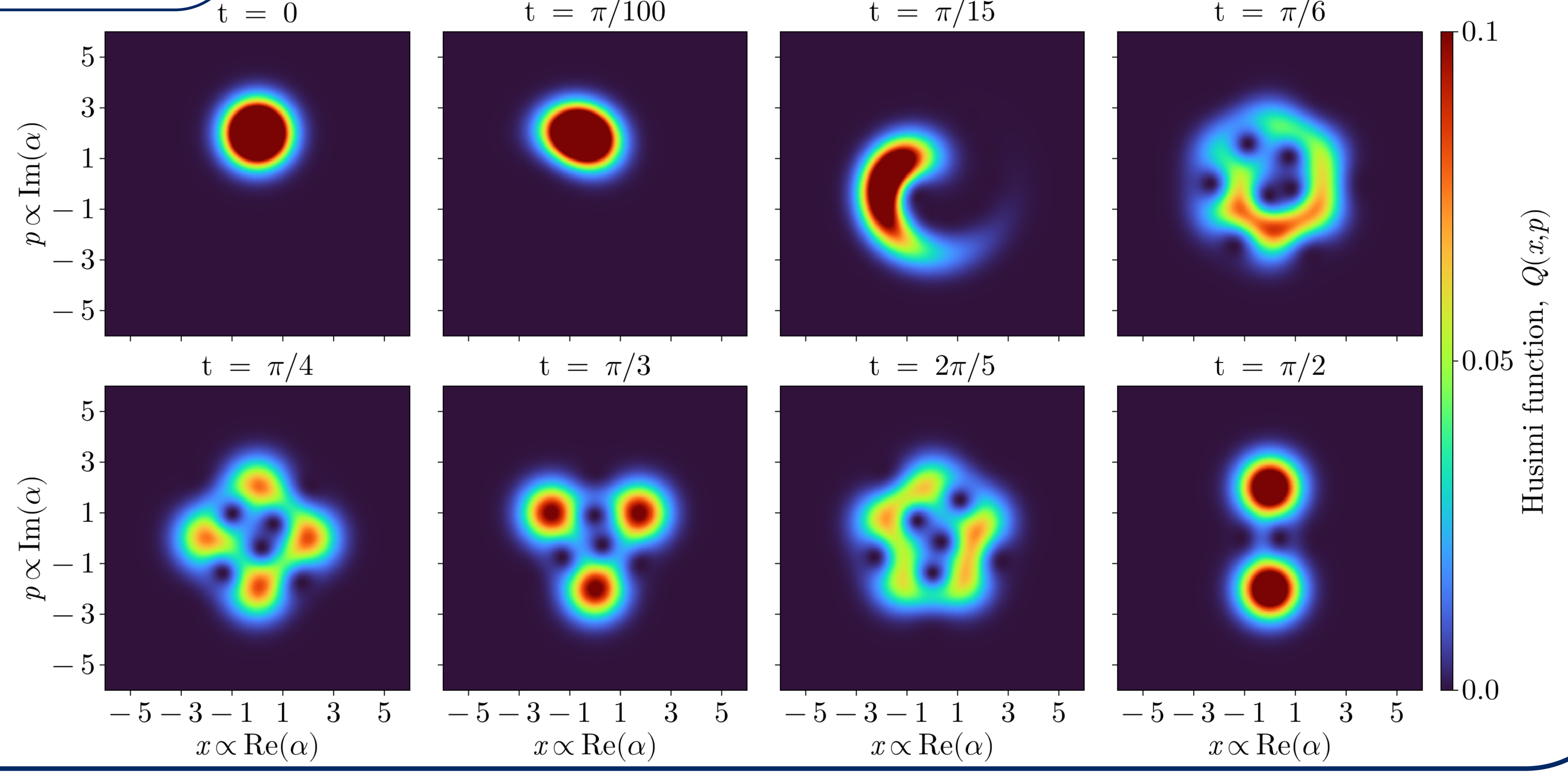
G. A. Hartmann Salvo, Master's thesis (Universidad de Chile, 2025).
S. Choi et al., Nat. Commun. 16, 7576 (2025).

$$W(q,p,t) = \frac{1}{\pi\hbar}\int dx \exp\left(\frac{-2ipx}{\hbar}\right)\psi^*(q-x,t)\psi(q+x,t)$$

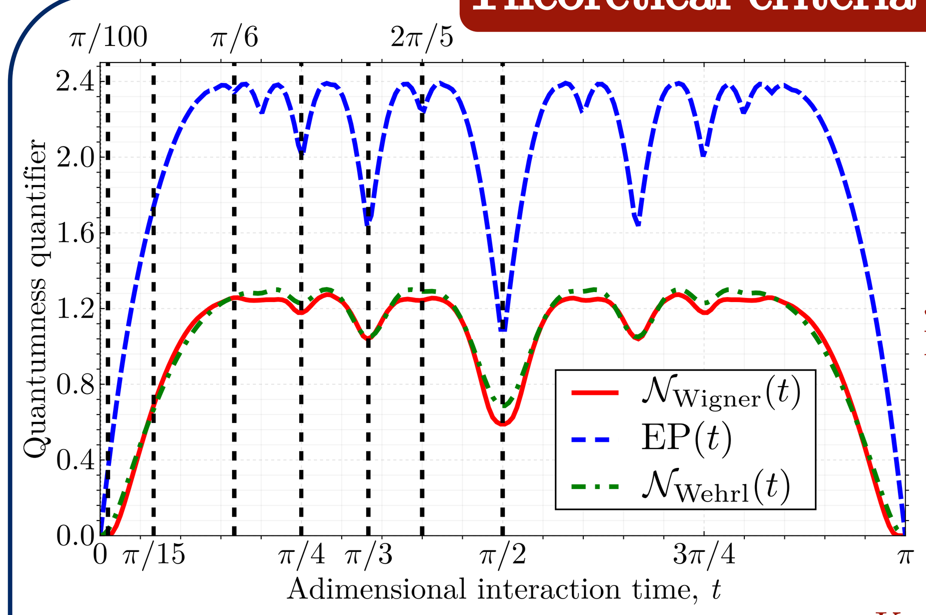
Characterization via full state tomography is experimentally demanding. Are there alternative metrics to identify quantum signatures in GCS that are more accessible to measure

$$Q(q,p,t) = \frac{1}{\pi\hbar}\iint dq'dp' \exp\left[-\frac{m\kappa(q'-q)^2}{\hbar} - \frac{(p'-p)^2}{\hbar m\kappa}\right]W(q',p',t)$$
$$= -\frac{1}{2\pi}\iint [\phi(\nu;q,p)]^*\rho(\mu,\nu,t)\phi(\mu;q,p)d\mu d\nu$$
$$\phi(\mu;q,p) = (\pi)^{-1/4}\exp\left[-\frac{1}{2}(\mu-q)^2 + ip\mu\right]$$





Theoretical criteria of non-classicality



Entanglement potential

$$EP(\sigma) = \log_2\left(\|(\hat\rho_\sigma)^{T_A}\|_1\right)$$
$$\hat\rho_\sigma = \hat U_{\rm BS}(\hat\sigma\otimes|0\rangle\langle 0|)(\hat U_{\rm BS})^\dagger$$
$$\hat\sigma_{\rm GCS} = |\alpha_{\varepsilon,t}\rangle\langle\alpha_{\varepsilon,t}|$$

J. K. Asbóth et al., Phys. Rev. Lett. 94, 173602 (2005)

$$\mathcal{N}_{\rm Wigner}(t) = \iint |W(q,p,t)|dqdp - 1$$

Wigner negativity

Kenfack and Życzkowski, J. Opt. B 6, 396 (2004).

Wehrl entropy non-classicality

$$S^{\rm Wehrl}_{\hat\rho}(t) = -\iint\frac{dqdp}{\pi}Q_{\hat\rho}(q,p,t)\ln[Q_{\hat\rho}(q,p,t)]$$
$$\mathcal{N}_{\rm Wehrl}(t) = \max\left\{0, S^{\rm Wehrl}_{\hat\rho}(t) - \sup_{\hat\sigma\in\Omega_{\rm classical}}[S^{\rm Wehrl}_{\hat\sigma}(t)]\right\}$$
$$\mathcal{N}^{\rm pure}_{\rm Wehrl}(t) = S^{\rm Wehrl}_{|\psi\rangle}(t) - 1$$

S. Bose, J. Phys. A 52, 025303 (2018)

Hillery-Zubairy criterion

Alternative N°1: Unitary rotation of the second mode

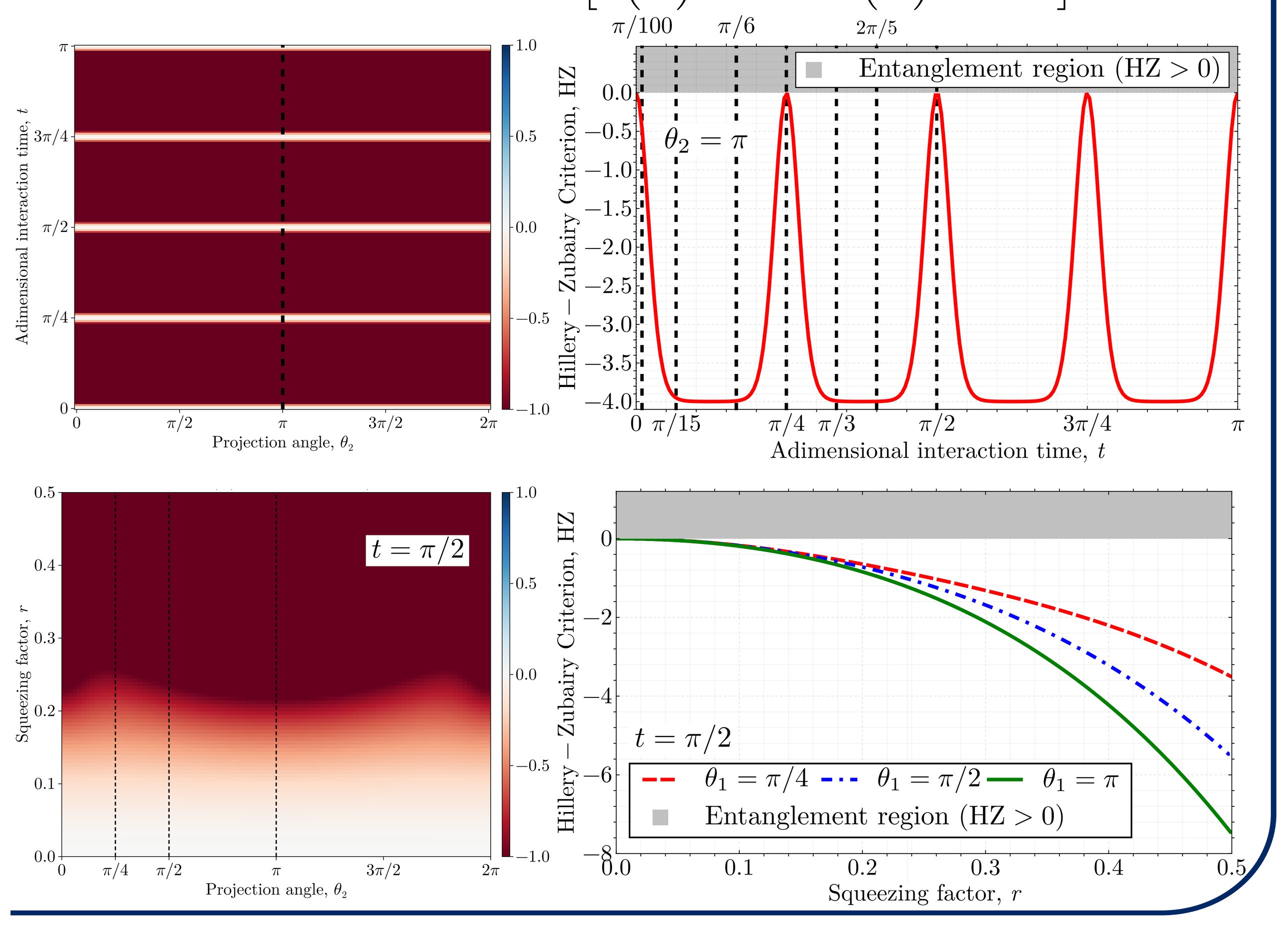
$$\hat a_2^{\rm after} = \hat a_2^{\rm before}\exp(i\theta_2)$$

Alternative N°2: Local squeezing operation (Bogoliubov transformation)

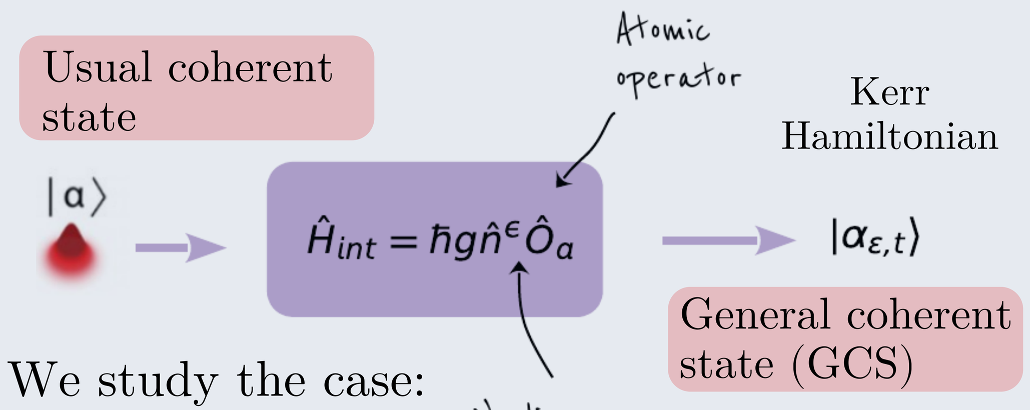
$$\hat a_2^{\rm before} = \cosh(r)\hat a_2^{\rm before} + \exp(i\theta_2)\sinh(r)\left(\hat a_2^{\rm before}\right)^\dagger$$

Hillery and Zubairy, Phys. Rev. Lett. 96, 050503 (2006).

$$\mathrm{HZ} = |\langle(\hat a_1)^m(\hat a_2)^n\rangle| - \left[\left\langle\left(\hat a_1^\dagger\right)^m(\hat a_1)^m\right\rangle\left\langle\left(\hat a_2^\dagger\right)^n(\hat a_2)^n\right\rangle\right]^{1/2}$$



Theoretical description of the state



We study the case:

$$|\alpha| = 2.0$$
$$\varepsilon = 2.0$$

$$|\alpha_{\varepsilon,t}\rangle = e^{-\frac{|\alpha|^2}{2}}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}e^{-itn^\epsilon}|n\rangle_F$$

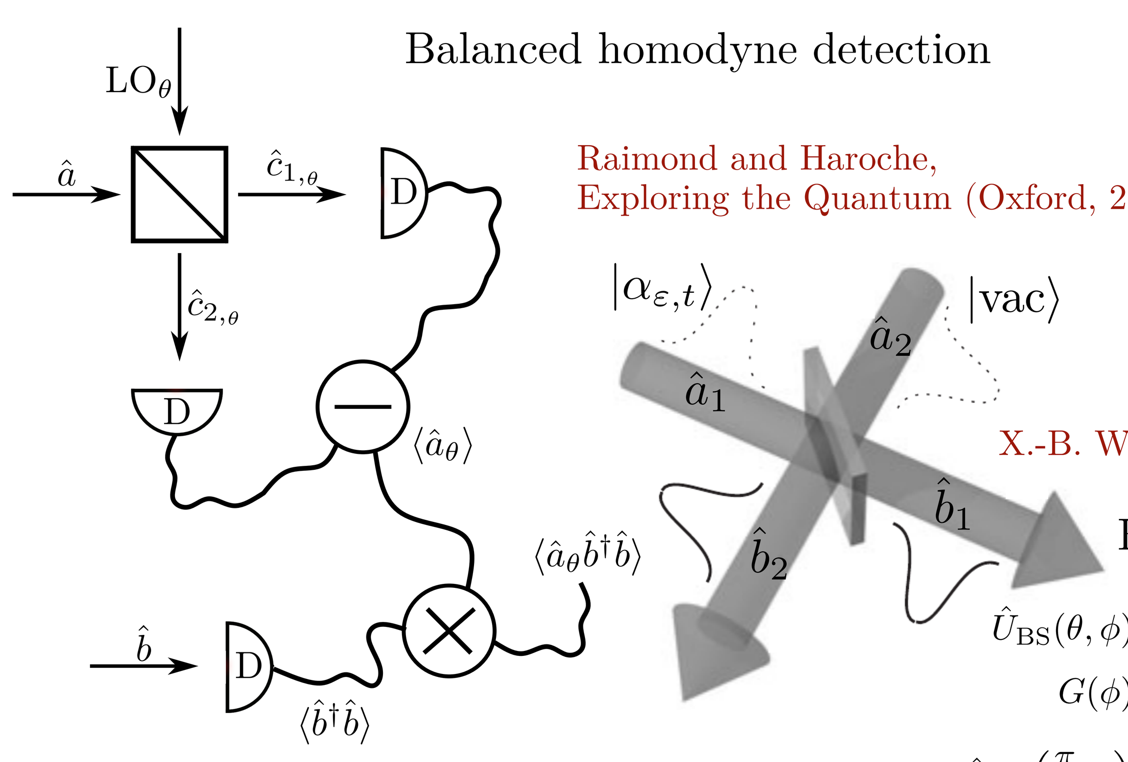
Nonlinear parameter; Effective interaction time; Fock basis

$$\hat\rho = \int\frac{d^2\alpha}{\pi}P(\alpha,\alpha^*)|\alpha\rangle\langle\alpha|$$

Glauber-Sudarshan P distribution

L. Mandel, Phys. Scr. T12, 34 (1986).

Experimental detection



Raimond and Haroche, Exploring the Quantum (Oxford, 2006).

X.-B. Wang, Phys. Rev. A 66, 024303 (2002).

Beam-Splitter transformation

$$\hat U_{\rm BS}(\theta,\phi) = \exp[-iG(\phi)\theta]$$
$$G(\phi) = -\left[e^{-i\phi}\hat b_1(\hat b_2)^\dagger + e^{-i\phi}(\hat b_1)^\dagger\hat b_2\right]$$
$$\text{(BS 50-50) } \hat U_{\rm BS}\left(\frac{\pi}{4},0\right) = \exp\left\{\frac{-i\pi}{4}\left[\hat b_1(\hat b_2)^\dagger + (\hat b_1)^\dagger\hat b_2\right]\right\}$$

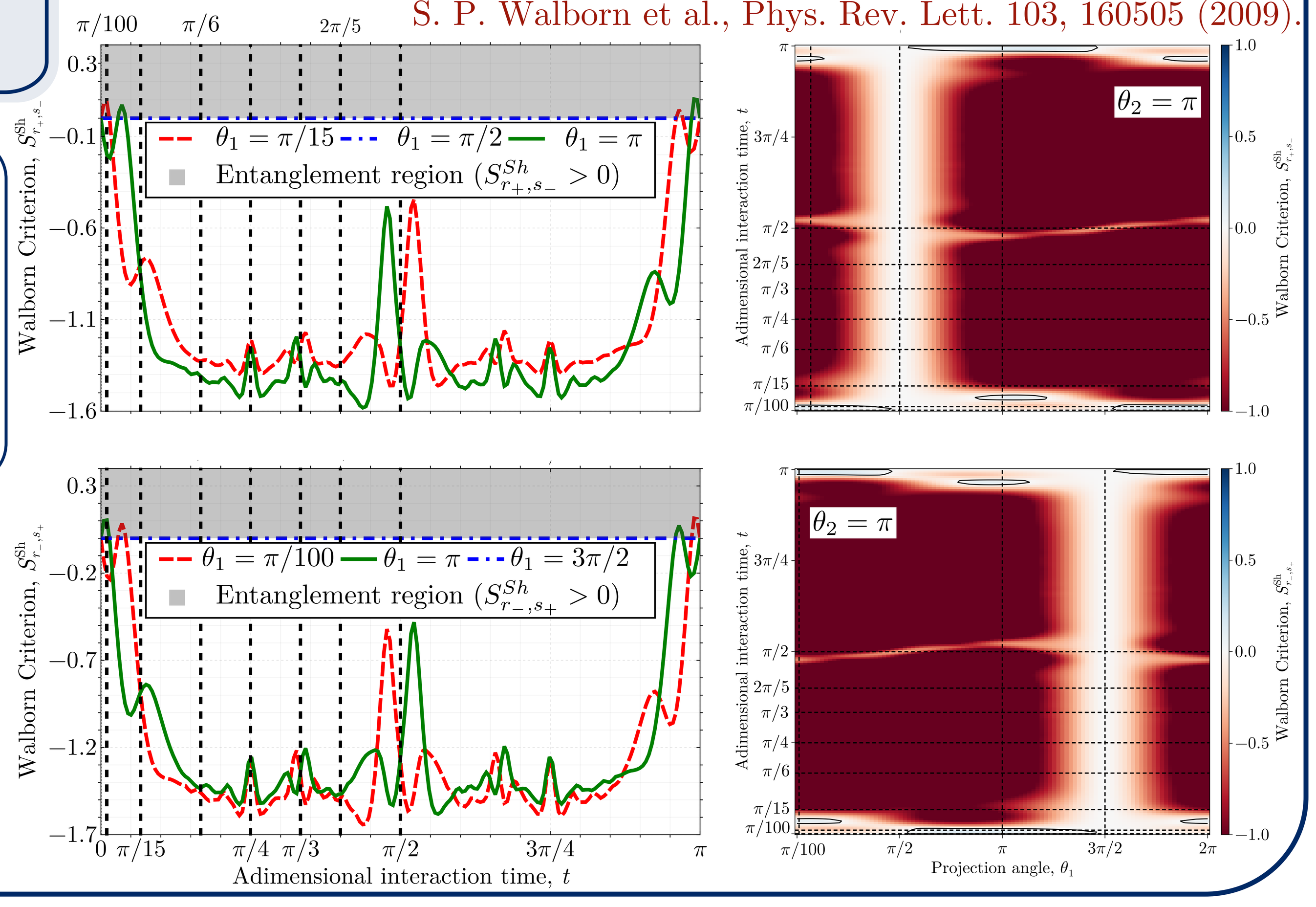
This criterion effectively identifies entanglement in squeezed states, but it does not hold for all times.

Walborn criterion

$$\rho(r_{\theta_1}, r_{\theta_2}) = ||\psi(r_{\theta_1}, r_{\theta_2})||^2$$
$$\eta(r_\pm) = \int\rho(r_{\theta_1}, \pm(r_\pm - r_{\theta_1}))dr_{\theta_1}$$
$$\eta(s_\mp) = \int\rho(s_{\theta_1}, \mp(s_\mp - s_{\theta_1}))ds_{\theta_1}$$
$$S^{\rm Sh}_{r_\pm} = -\int\eta(r_\pm)\ln[\eta(r_\pm)]dr_\pm$$
$$S^{\rm Sh}_{r_\pm,s_\mp} = \ln(2\pi e) - S^{\rm Sh}_{r_\pm} + S^{\rm Sh}_{s_\mp}$$

S. P. Walborn et al., Phys. Rev. Lett. 103, 160505 (2009).

$$\hat r_\pm = \hat r_{\theta_1} \pm \hat r_{\theta_2}$$
$$\hat s_\pm = \hat r_{(\theta_1+\pi/2)} \pm \hat r_{(\theta_2+\pi/2)}$$
$$\hat r_\theta = \frac{1}{\sqrt2}\left(e^{-i\theta}\hat a + e^{i\theta}\hat a^\dagger\right)$$



Additional information

I. S. Valdivieso et al., arXiv:2512.15655 (2025).
G. Kirchmair et al., Nature 495, 205 (2013).
M. Uria et al., Phys. Rev. Res. 5, 013165 (2023).

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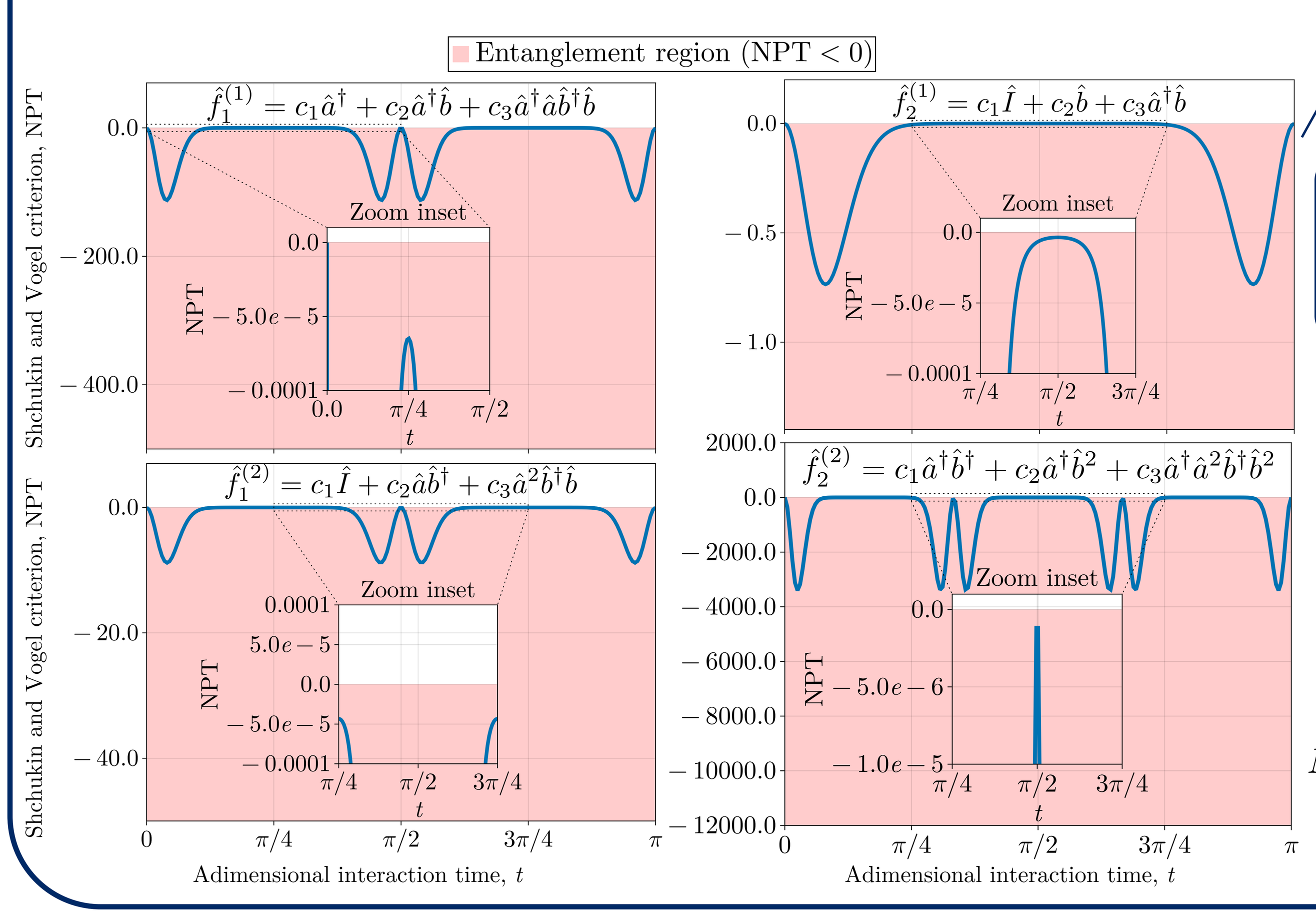
QuantumToolbox.jl

Shchukin and Vogel criterion

E. Shchukin and W. Vogel, Phys. Rev. Lett. 95, 230502 (2005).



$$s = \begin{vmatrix} 1 & \langle\hat b^\dagger\rangle & \langle\hat a\hat b^\dagger\rangle \\ \langle\hat b\rangle & \langle\hat b^\dagger\hat b\rangle & \langle\hat a\hat b^\dagger\hat b\rangle \\ \langle\hat a^\dagger\hat b\rangle & \langle\hat a^\dagger\hat b^\dagger\hat b\rangle & \langle\hat a^\dagger\hat a\hat b^\dagger\hat b\rangle\end{vmatrix}$$

We identify families of first- and second-order correlators that serve as universal witnesses of quantumness.

Negativity of the partial transposition (NPT)

$$\langle\hat f^\dagger\hat f\rangle^{\rm PT} = \sum_{\substack{n,m,k,l\\p,q,r,s=0}}^{\infty} c^*_{pqrs}c_{nmkl}M_{pqrs,nmkl} \geqslant 0$$
$$\hat f = \sum_{n,m,k,l=0}^{\infty} c_{nmkl}\left(\hat a^\dagger\right)^n\hat a^m\left(\hat b^\dagger\right)^k\hat b^l$$
$$M_{pqrs,nmkl} = \left\langle\left(\hat a^\dagger\right)^q\hat a^p\left(\hat a^\dagger\right)^n\hat a^m\left(\hat b^\dagger\right)^l\hat b^k\left(\hat b^\dagger\right)^r\hat b^s\right\rangle$$

Peres-Horodecki condition

Conclusion and future perspectives

In this work, a detailed characterization of Generalized Coherent States (GCS) was achieved through an approach that prioritizes experimental viability over abstract theoretical metrics. Entanglement criteria based on two-mode correlators were proposed, the implementation of which is feasible via balanced homodyne detection and quantum state tomography, thereby overcoming the cost and complexity limitations associated with other non-classicality measures. Furthermore, a comparative analysis among various criteria allowed for the identification of their regions of validity and the hierarchical ranking of their sensitivity, establishing a solid foundation for the detection of quantum resources in real optical systems.

As future work, we propose extending this analysis to diverse families of GCS to determine the universality and robustness of the Shchukin-Vogel criteria under different configurations. Of particular interest is the adaptation of these formalisms to include statistical mixtures of GCS, which will enable the modeling of entanglement behavior in the presence of decoherence and thermal noise. Finally, we will explore the optimization of high-fidelity algorithms for the reconstruction of moment matrices, aiming to integrate high-performance computing methods to facilitate data processing in higher-dimensional systems.